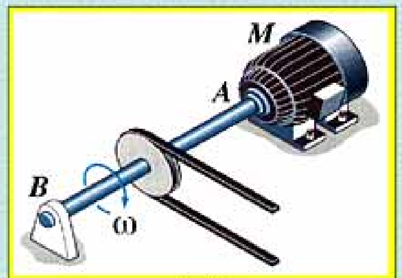
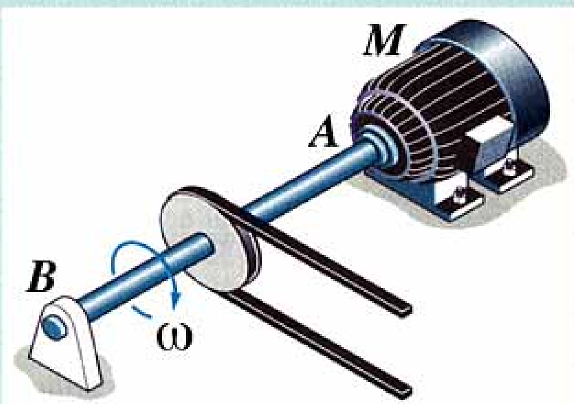
6. Torsion of Prismatic Bars

Examples of Members which Transmit Torque

- Propeller shafts
- Torque tubes of power equipment



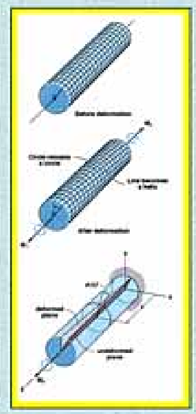
animation

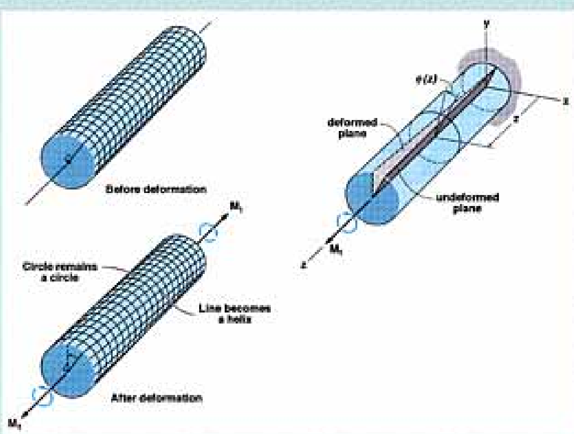


Bars with Circular Cross Sections

Basic Assumptions

- Plane parallel cross sections remain plane and parallel after deformation
- Cross sections of the bar rotate as rigid bodies about the z-axis
- Shearing strain varies linearly in the radial direction





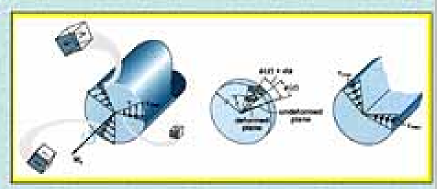
Bars with Circular Cross Sections

Kinematic Relations

$$\gamma = r \frac{d\phi}{dz}$$

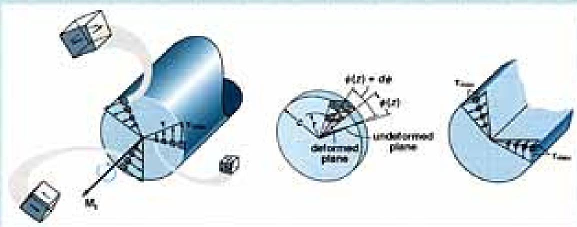
$$\theta = \frac{d\phi}{dz}$$

= angle of twist per unit length



Static Relations

$$M_t = \int_A \tau \, dA \, r$$



Bars with Circular Cross Sections

Constitutive Relations

$$\tau = G\,\gamma$$

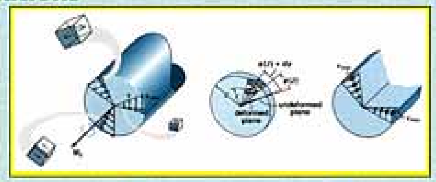
$$= G\,r\,\frac{d\phi}{dz}$$

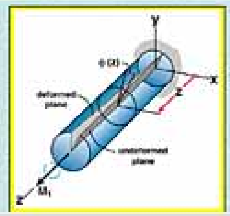
$$M_t = G\,\frac{d\phi}{dz}\,I_p$$

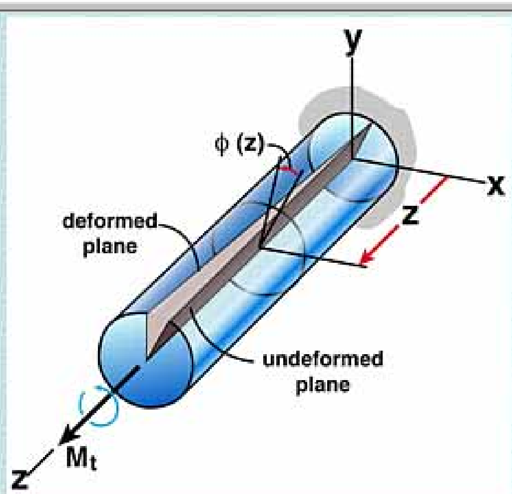
or

$$\tau = \frac{M_t}{I_p}\,r$$

$$\theta = \frac{M_t}{G\,I_p}$$



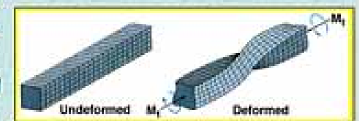




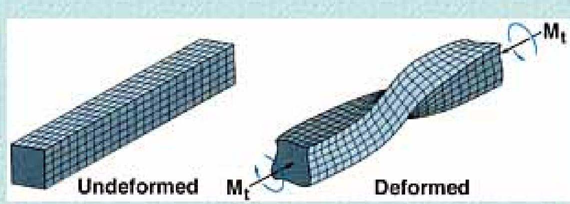
Bars with Solid Noncircular Cross Sections

Basic Assumptions

- Plane cross sections do not remain plane after deformation - they become warped surfaces. Warping is accompanied by increase in shear strain (and stress) in some parts and decrease in others.
- Cross sections do not distort in their own plane.
- Every point in the cross section rotates about a center of twist.



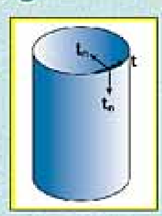
animation

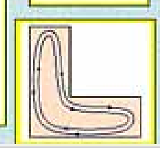


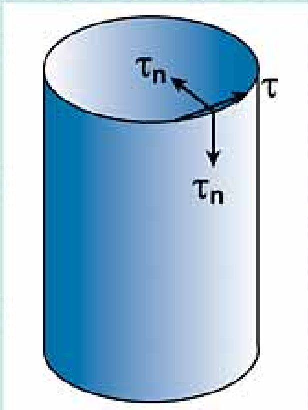
Bars with Solid Noncircular Cross Sections

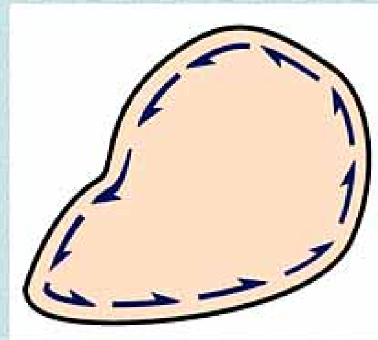
- Cross sections do not distort in their own plane.
- Every point in the cross section rotates about a center of twist.
- No external constraints exist to prevent any cross section from warping (Saint-Venant torsion).

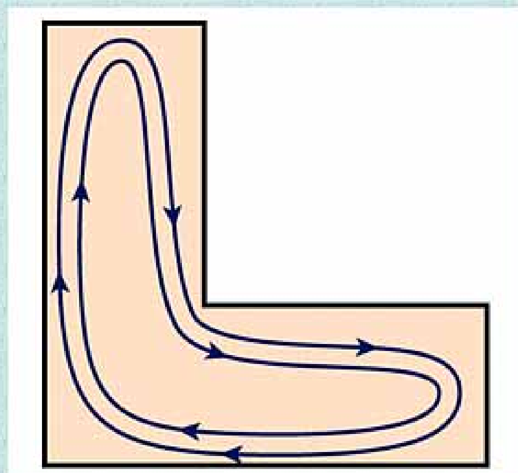
animation









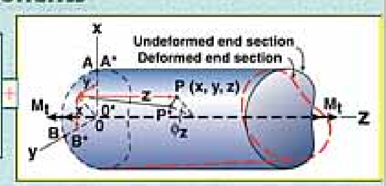


Bars with Solid Noncircular Cross Sections

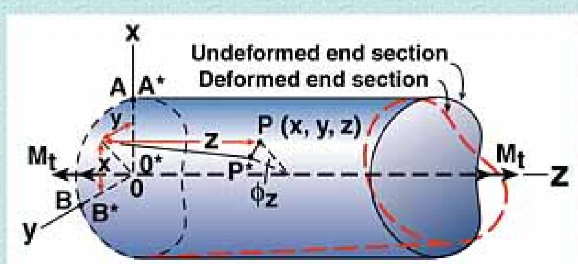
Kinematic Relations

- **Displacement components**

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \begin{Bmatrix} -yz \\ xz \\ \psi(x,y) \end{Bmatrix} \theta$$



where ψ is the warping function and q is the angle of twist per unit length.



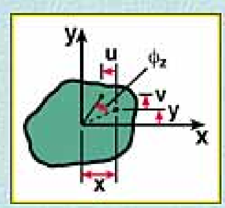
Bars with Solid Noncircular Cross Sections

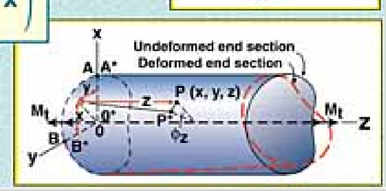
- **Strain components**

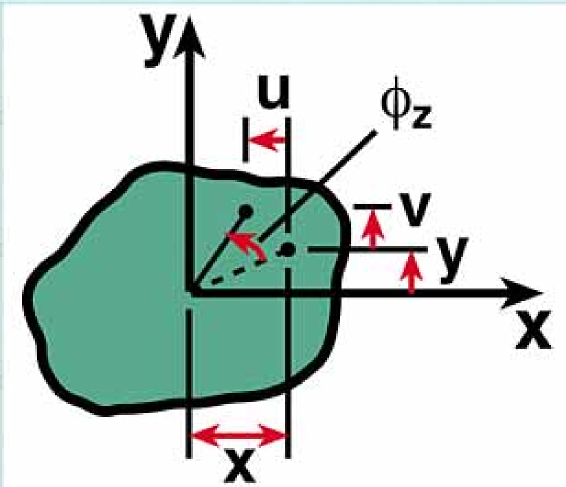
$$\varepsilon_x = \varepsilon_y = \varepsilon_z = 0$$

$$\gamma_{xy} = 0$$

$$\begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \theta \begin{Bmatrix} \dfrac{\partial \psi}{\partial x} - y \\ \dfrac{\partial \psi}{\partial y} + x \end{Bmatrix}$$



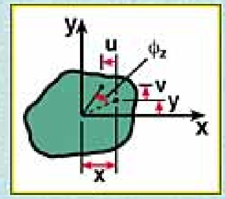


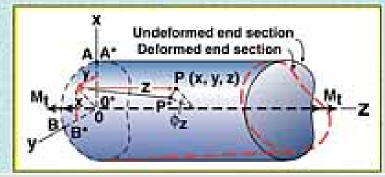


Bars with Solid Noncircular Cross Sections

- *Strain components satisfy the following compatibility equation*

$$\frac{\partial \gamma_{xz}}{\partial y} - \frac{\partial \gamma_{yz}}{\partial x} = -2\theta$$



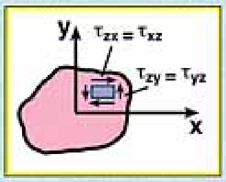


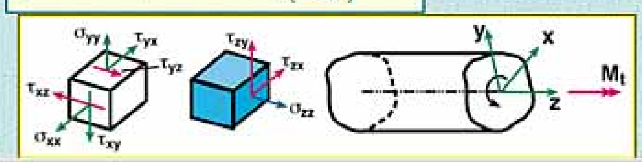
Bars with Solid Noncircular Cross Sections

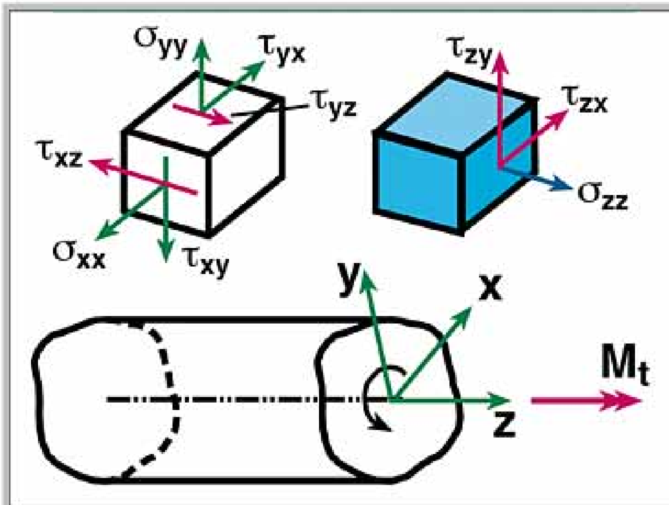
Static Relations

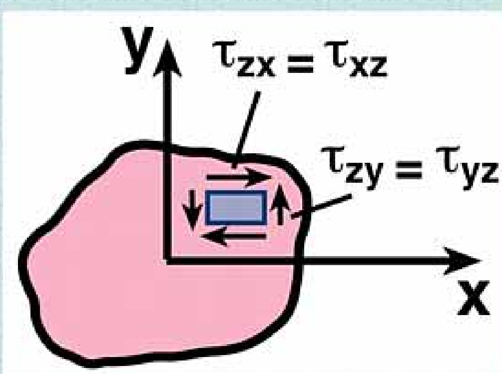
Equilibrium Equations

$$\begin{bmatrix} \frac{\partial}{\partial x} & \cdot & \cdot & \cdot & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ \cdot & \frac{\partial}{\partial y} & \cdot & \frac{\partial}{\partial z} & \cdot & \frac{\partial}{\partial x} \\ \cdot & \cdot & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & \cdot \end{bmatrix} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = 0$$









Bars with Solid Noncircular Cross Sections

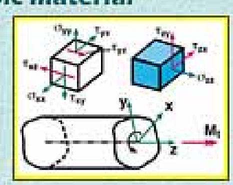
Constitutive Relations

For linearly elastic isotropic material

$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = 0$$

$$\tau_{xy} = 0$$

$$\begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = G \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}$$



Bars with Solid Noncircular Cross Sections

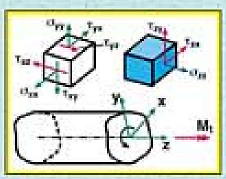
Therefore, the equilibrium equations reduce to:

$$\frac{\partial \tau_{xz}}{\partial z} = 0 \;\rightarrow\; \tau_{xz} = \tau_{xz}(x,y)$$

$$\frac{\partial \tau_{yz}}{\partial z} = 0 \;\rightarrow\; \tau_{yz} = \tau_{yz}(x,y)$$

and

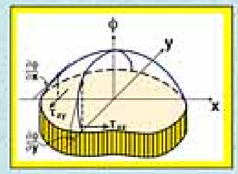
$$\frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{xz}}{\partial x} = 0$$



Bars with Solid Noncircular Cross Sections

- **Introducing a stress function $\phi = \phi(x,y)$ such that**

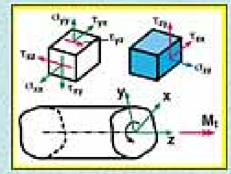
$$\begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial \phi}{\partial y} \\ -\dfrac{\partial \phi}{\partial x} \end{Bmatrix}$$

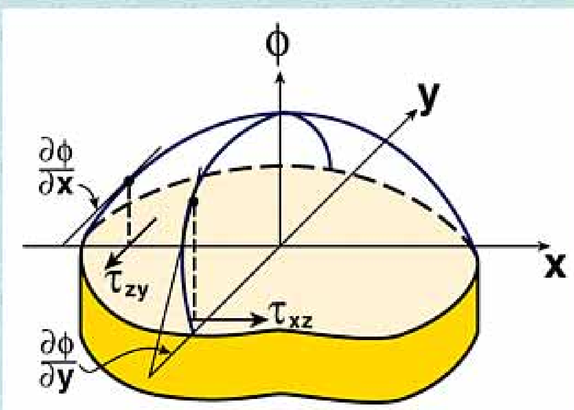


The equilibrium equation

$$\frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{xz}}{\partial x}$$

becomes identically satisfied.



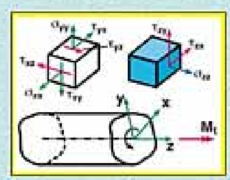


Bars with Solid Noncircular Cross Sections

- **The compatibility equation** $\dfrac{\partial \gamma_{xz}}{\partial y} - \dfrac{\partial \gamma_{yz}}{\partial x} = -2\theta$

becomes $\dfrac{1}{G}\left(\dfrac{\partial^2 \phi}{\partial x^2} + \dfrac{\partial^2 \phi}{\partial y^2}\right) = -2\theta$

or $\nabla^2 \phi = -2G\theta$



Bars with Solid Noncircular Cross Sections

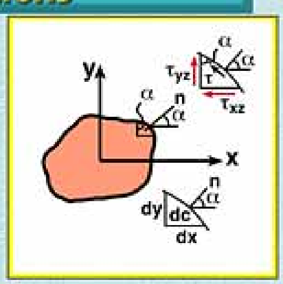
Boundary Conditions

$$\sin\alpha = \frac{dx}{dc}$$

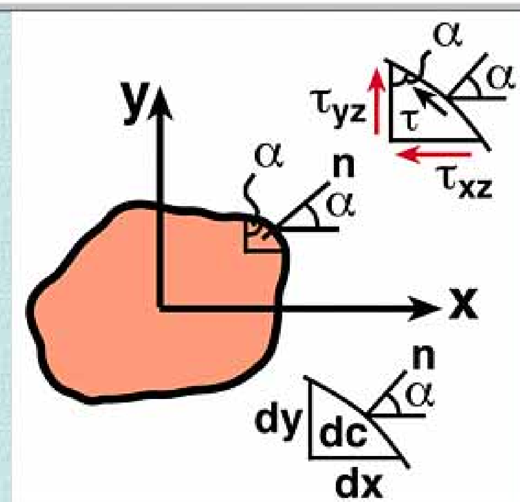
$$\cos\alpha = \frac{dy}{dc}$$

$$\tau_{yz} = \tau\cos\alpha$$

$$\tau_{xz} = \tau\sin\alpha$$



Since the shear stresses are tangential to the boundary $\tau_{yz}\sin\alpha - \tau_{xz}\cos\alpha = 0$



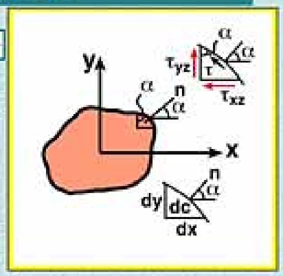
Bars with Solid Noncircular Cross Sections

$$\tau_{yz} \sin\alpha - \tau_{xz} \cos\alpha = 0$$

or

$$\left(-\frac{\partial\phi}{\partial x}\right)\frac{dx}{dc} - \left(\frac{\partial\phi}{\partial y}\right)\frac{dy}{dc} = 0$$

$$\frac{d\phi}{dc} = 0$$



or, ϕ = const. at the boundary

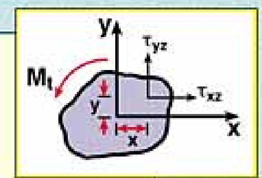
Bars with Solid Noncircular Cross Sections

Twisting Moment

$$M_t = \int_A (-\tau_{xz}\, y + \tau_{yz}\, x)\,dA$$

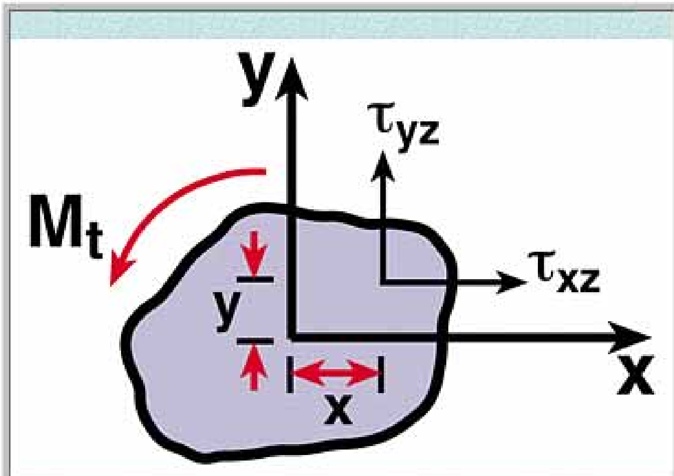
$$= \int_A \left(-\frac{\partial\phi}{\partial y}\, y + \left(-\frac{\partial\phi}{\partial x}\right) x\right) dA$$

$$= \int_A 2\phi\, dA + \oint_c \left[y\left(-\frac{dx}{dc}\right) + x\left(-\frac{dy}{dc}\right)\right]\phi\, dc$$



If ϕ is set equal to zero at the boundary, then

$$M_t = \int_A 2\phi\, dA$$



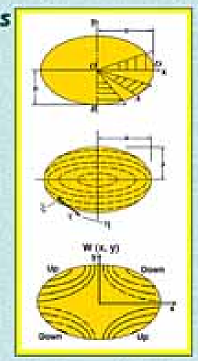
Torsion Formulas for Special Cross Sections

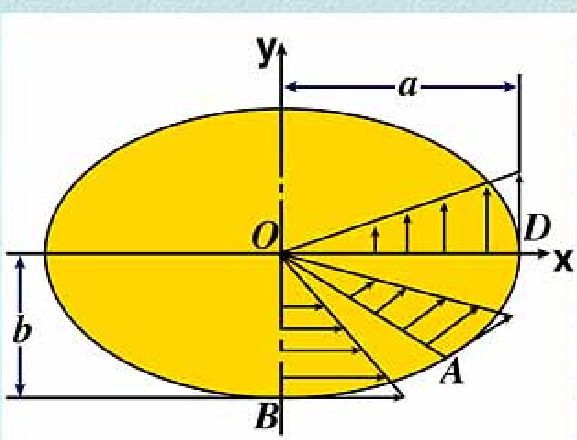
Elliptic Cross Sections

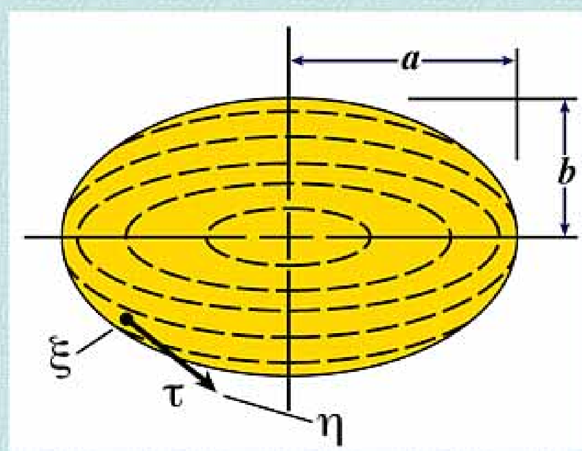
Stress Function and Shear Stresses

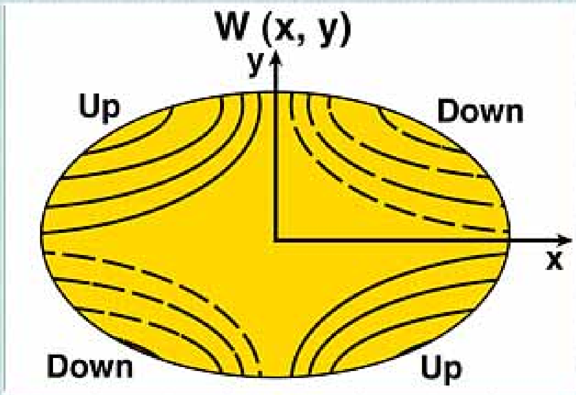
$$\phi = -\frac{M_t}{\pi ab}\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right)$$

$$\begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = \frac{2M_t}{\pi ab}\begin{Bmatrix} -\dfrac{y}{b^2} \\ \dfrac{x}{a^2} \end{Bmatrix}$$







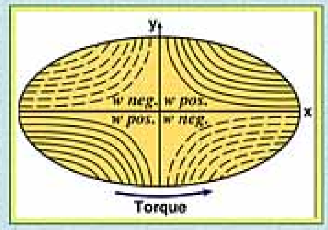


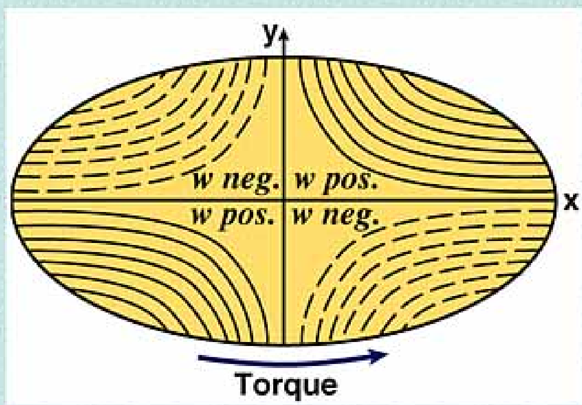
Torsion Formulas for Special Cross Sections

Elliptic Cross Sections

Axial Displacement

$$w = \frac{M_t}{\pi a^3 b^3 G}\left(b^2 - a^2\right) xy$$



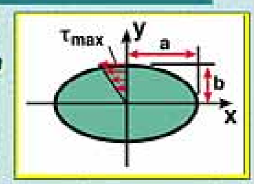


Torsion Formulas for Special Cross Sections

Elliptic Cross Sections

- ***Angle of twist per unit length***

$$\theta = \frac{M_t}{G I_t}$$



where

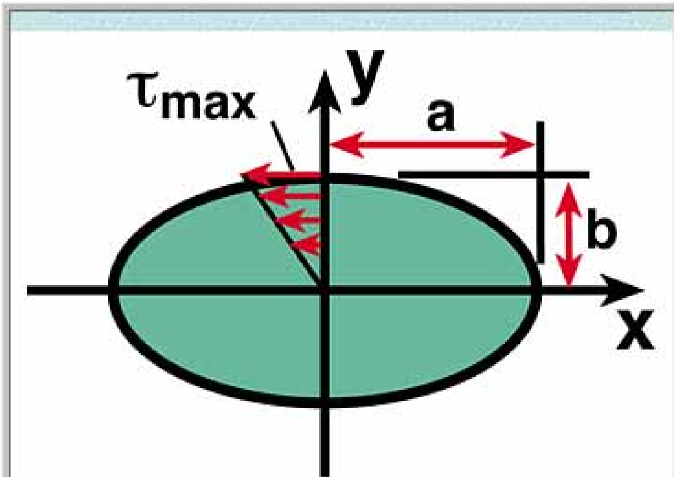
$$I_t = \pi a^3 b^3 / \left(a^2 + b^2\right)$$

I_t is called the torsion constant.

$G I_t$ is the torsional stiffness.

- ***Maximum shear stress occurs at the boundary nearest the centroid of the cross section***

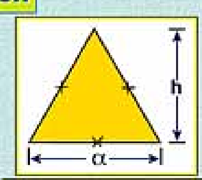
$$\tau_{max} = \frac{M_t}{\pi a b^2 / 2}$$



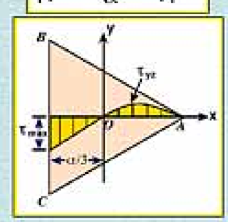
Torsion Formulas for Special Cross Sections

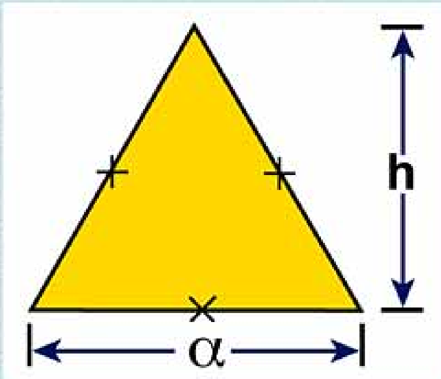
Equilateral Triangular Cross Section

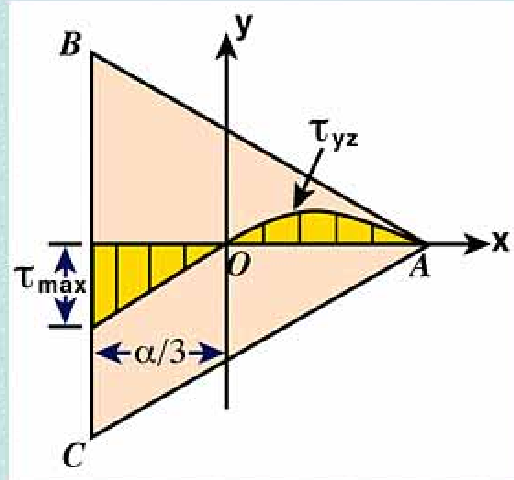
$$\theta = \frac{M_t}{G I_t}$$



$$I_t = \frac{h^4}{15\sqrt{3}}$$





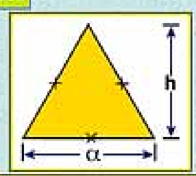


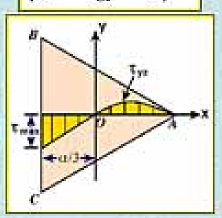
Torsion Formulas for Special Cross Sections

Equilateral Triangular Cross Section

$$I_t = \frac{h^4}{15\sqrt{3}}$$

$$= \frac{\sqrt{3}}{80}\,a^4 = \frac{a^4}{46.188}$$



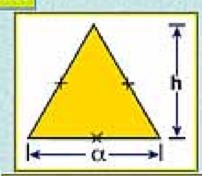


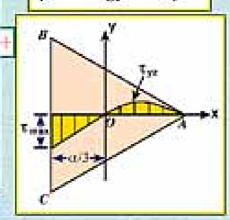
Torsion Formulas for Special Cross Sections

Equilateral Triangular Cross Section

$$\tau_{max} = \frac{M_t}{2\,h^3 / \left(15\sqrt{3}\right)}$$

$$= \frac{M_t}{a^3 / 20}$$



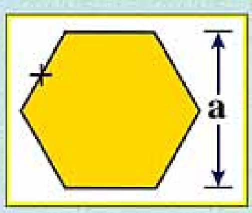


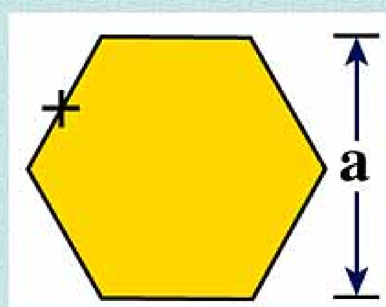
Torsion Formulas for Special Cross Sections

Hexagonal Cross Sections

$$\theta = \frac{M_t}{G\,I_t}$$

$$I_t = \frac{a^4}{8.8}$$

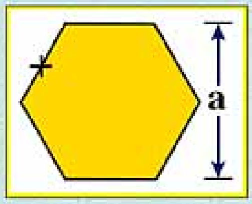




Torsion Formulas for Special Cross Sections

Hexagonal Cross Sections

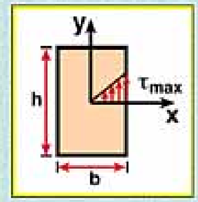
$$I_t = \frac{a^4}{8.8}$$



$$\tau_{max} = \frac{M_t}{\left(a^3 / 5.7\right)}$$

Torsion Formulas for Special Cross Sections

Rectangular Cross Section



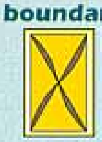
- *Angle of twist per unit length*

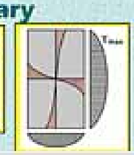
$$\theta = \frac{M_t}{G I_t}$$

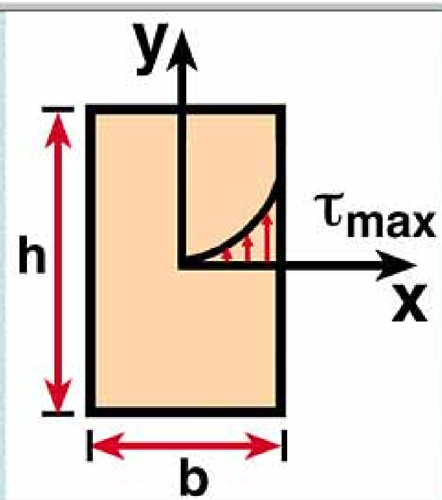
where $I_t = k_1 h b^3$

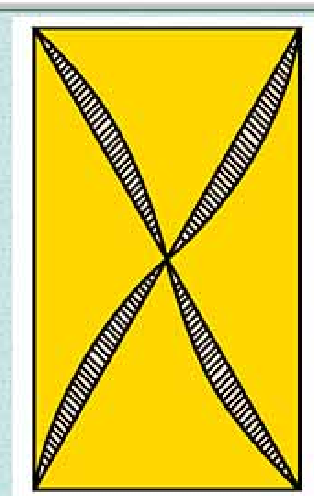
- *Maximum shear stresses occur at the center of the long side at the boundary*

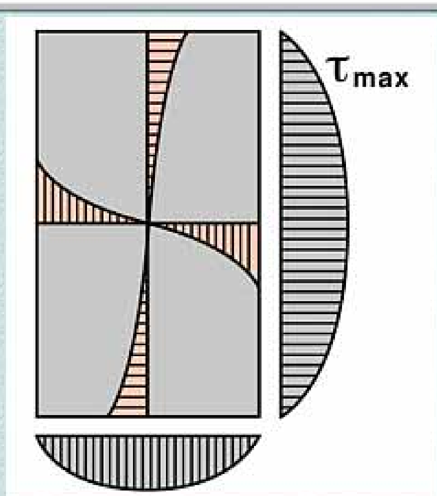
$$\tau_{max} = \frac{M_t}{k_2 h b^2}$$





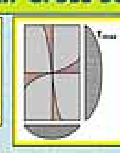


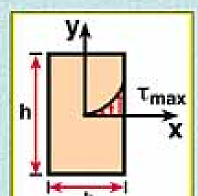




Torsion Formulas for Special Cross Sections

Rectangular Cross Section





k_1 and k_2 are parameters which depend on the ratio h/b.

h/b	1	1.5	2	2.5	...	10	∞
k_1	0.141	0.196	0.229	0.249		0.312	0.333
k_2	0.208	0.231	0.246	0.256		0.312	0.333

Torsion Formulas for Special Cross Sections

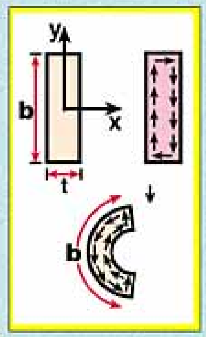
Narrow Rectangular Sections

$$\tau_{max} = \frac{M_t t}{I_t}$$

$$\theta = \frac{M_t}{G I_t}$$

where

$$I_t \simeq \frac{1}{3} b t^3$$



Torsion Formulas for Special Cross Sections

Narrow Rectangular and Thin-Walled Cross Sections

$$\tau_{max} = \frac{M_t (t_i)_{max}}{I_t}$$

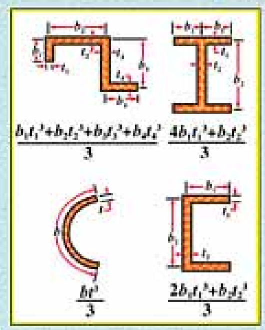
$$\theta = \frac{M_t}{G I_t}$$

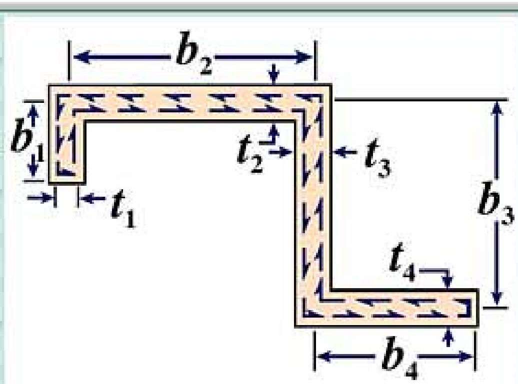
where

$$I_t = \alpha \sum_i \frac{1}{3} b_i t_i^3$$

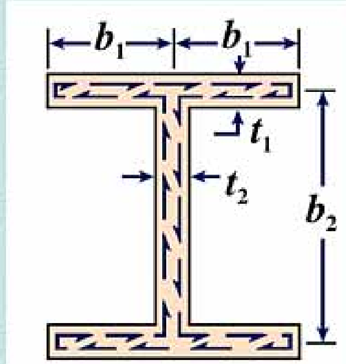
α is a correction factor.

If all $b_i > 10\, t_i$, then $\alpha = 1$.

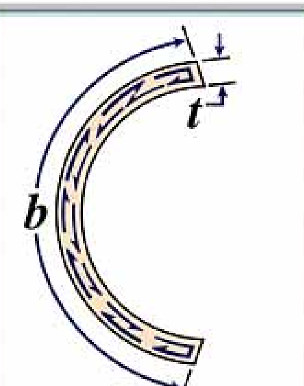




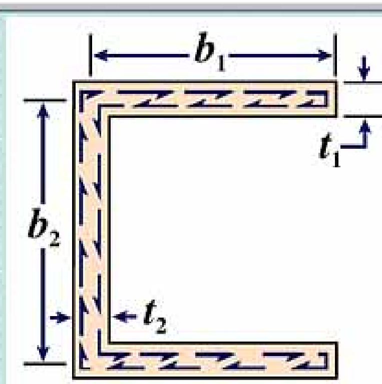
$$\frac{b_1 t_1^3 + b_2 t_2^3 + b_3 t_3^3 + b_4 t_4^3}{3}$$



$$\frac{4 b_1 t_1^3 + b_2 t_2^3}{3}$$



$$\frac{b t^3}{3}$$

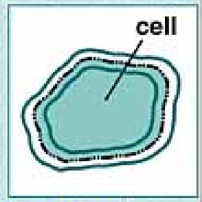


$$\frac{2 b_1 t_1^3 + b_2 t_2^3}{3}$$

Bars with Thin-walled Closed Cross Sections

Definitions

- **Tubes – thin-walled closed sections**
- **Cell – area enclosed by a tube**

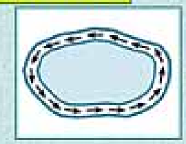


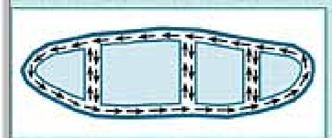
Classification of Thin-Walled Closed Sections

Bars with Thin-walled Closed Cross Sections

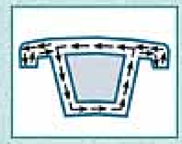
Classification of Thin-Walled Closed Sections

- **Single cell tube – encloses only one cell**





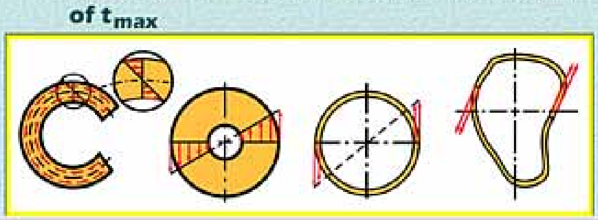
- **Multicell tube – encloses more than one cell**
- **Hybrid tube – composed of a closed cell plus open fin elements (mixed open-closed cross section)**



Bars with Thin-walled Closed Cross Sections

Difference Between Torsional Shear Stresses in Bars with Open and Closed Sections

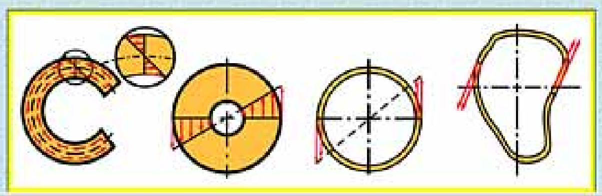
- **Open thin-walled section**
  - **Shear stresses are linear through the thickness and are zero at the centerline**
  - **Maximum shear stresses occur at location of t_{max}**



Bars with Thin-walled Closed Cross Sections

Difference Between Torsional Shear Stresses in Bars with Open and Closed Sections

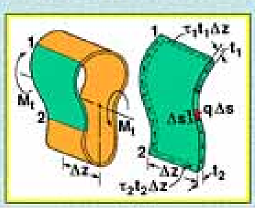
- **Closed hollow circular section**
  - **Shear stresses vary linearly with the radius r, becoming nearly uniform for very thin tubes**

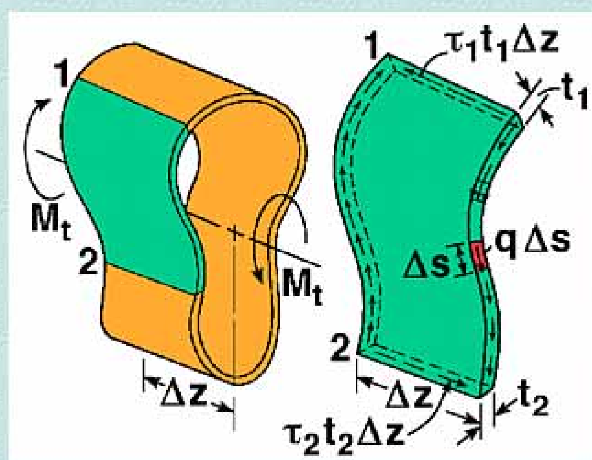


Bars with Thin-walled Closed Cross Sections

Shear Stresses and Shear Flow in Thin-Walled Closed Sections (Tubes)

- **Shear stresses are tangent to the wall of the cross section**
- **Shear flow at any point is the product of the shearing stress and the thickness**
  $q = \tau t$
- **Equilibrium of a slice of the bar**
  - **For any cross section** $q = \tau_1 t_1 = \tau_2 t_2$ **i.e., shear flow is constant.**





Bars with Thin-walled Closed Cross Sections

- **Equilibrium of a slice of the bar**
  - **For any cross section**
    $q = \tau_1 t_1 = \tau_2 t_2$
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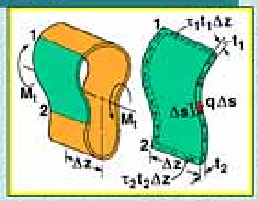
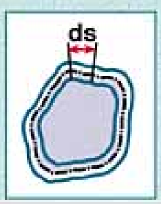


  - **Relation between twisting moment and shear flow**

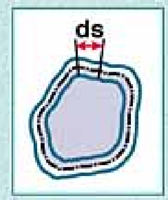
$$M_t = \oint q\, r\, ds = q \oint r\, ds$$



Bars with Thin-walled Closed Cross Sections

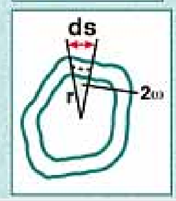
- **Relation between twisting moment and shear flow**

$$M_t = \oint q\, r\, ds = q \oint r\, ds$$



- **The integral**

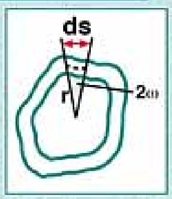
$$\int_0^{s_1} r\, ds = 2\omega(s_1)$$



Bars with Thin-walled Closed Cross Sections

- **The integral**

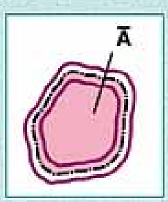
$$\int_0^{s_1} r\, ds = 2\omega(s_1)$$



where ω is the sectorial area.

- **For the entire cross section**

$$\oint r\, ds = 2\bar{A}$$



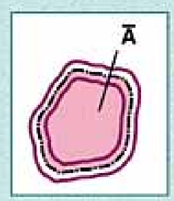
where $\bar{A}$ is the total area enclosed by the centerline of the tube.

$$M_t = q\, 2\bar{A}$$

Bars with Thin-walled Closed Cross Sections

- **For the entire cross section**

$$\oint r\, ds = 2\bar{A}$$



where $\bar{A}$ is the total area enclosed by the centerline of the tube.

$$M_t = q\, 2\bar{A}$$

or

$$q = \tau t = \frac{M_t}{2\bar{A}}$$

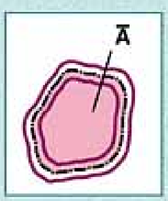
Bars with Thin-walled Closed Cross Sections

where $\bar{A}$ is the total area enclosed by the centerline of the tube.

$$M_t = q\, 2\bar{A}$$

or

$$q = \tau t = \frac{M_t}{2\bar{A}}$$



- ***Maximum shear stresses occur at t_{min} (by contrast, in open thin-walled section they occur at t_{max}).***

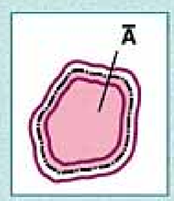
Bars with Thin-walled Closed Cross Sections

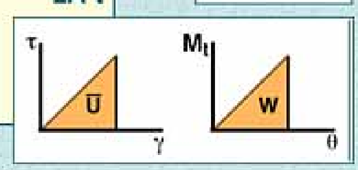
Torsional Strain Energy per Unit Length

Torsional shearing strain $\gamma = \dfrac{\tau}{G}$

$$U = \frac{1}{2}\oint_0^s \tau\gamma\, t\, ds = \frac{1}{2}\oint_0^s \frac{\tau^2}{G}\, t\, ds \ , \quad \tau = \frac{M_t}{2\bar{A}\, t}$$

$$U = \frac{M_t^2}{8\bar{A}^2 G}\oint_0^s \frac{ds}{t}$$





Bars with Thin-walled Closed Cross Sections

Work Done by the Twisting Moment (per Unit Length)

$$W = \frac{1}{2} M_t \theta = U$$

Angle of Twist per Unit Length

$$\theta = \frac{M_t}{G I_t} \quad \text{where} \quad I_t = 4\bar{A}^2 \Big/ \oint \frac{ds}{t}$$

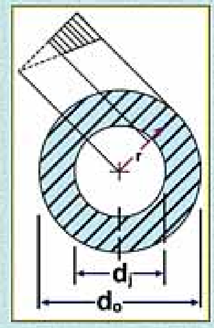
Bars with Thin-walled Closed Cross Sections

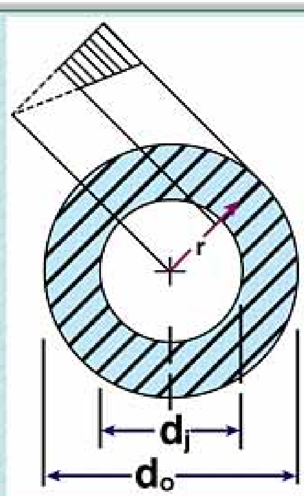
Comparison Between the Torsion Formulas for Circular Cross Section and Those Tubes

Circular Cross Section

$$\tau_{max} = \frac{M_t r_{max}}{I_p}$$

$$\theta = \frac{M_t}{G I_p}$$





Bars with Thin-walled Closed Cross Sections

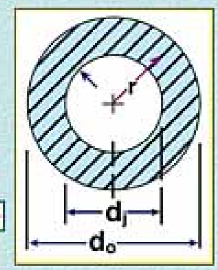
Comparison Between the Torsion Formulas for Circular Cross Section and Thin Tube

Circular Cross Section

where

$$r_{max} = \frac{d_o}{2}$$

$$I_p = \frac{\pi}{32}\left(d_o^4 - d_i^4\right)$$



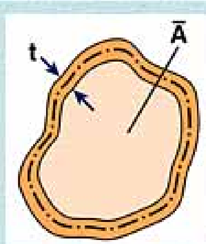
Bars with Thin-walled Closed Cross Sections

Comparison Between the Torsion Formulas for Circular Cross Section and Those Tubes

Single-Cell Tube

$$\tau_{max} = \frac{M_t}{2\bar{A} t_{min}}$$

$$\theta = \frac{M_t}{G I_t}$$



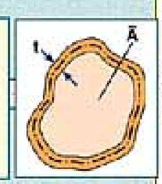
Bars with Thin-walled Closed Cross Sections

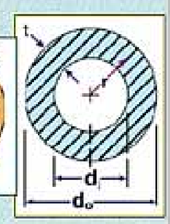
Comparison Between the Torsion Formulas for Circular Cross Section and Those Tubes

Single-Cell Tube

where

$$\bar{A} = \frac{\pi}{4}\left(\frac{d_o + d_i}{2}\right)^2$$





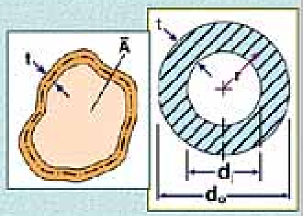
Bars with Thin-walled Closed Cross Sections

Comparison Between the Torsion Formulas for Circular Cross Section and Those Tubes

Single-Cell Tube

where

$$t = \frac{1}{2}\left(d_o - d_i\right)$$



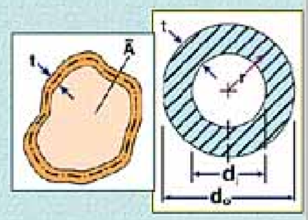
Bars with Thin-walled Closed Cross Sections

Comparison Between the Torsion Formulas for Circular Cross Section and Those Tubes

Single-Cell Tube

where

$$I_t = \frac{4\,\bar{A}^2}{\oint \frac{ds}{t}}$$



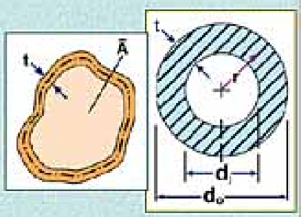
Bars with Thin-walled Closed Cross Sections

Comparison Between the Torsion Formulas for Circular Cross Section and Those Tubes

Single-Cell Tube

where

$$= \frac{4\,\bar{A}^2}{\frac{1}{t}\oint ds}$$



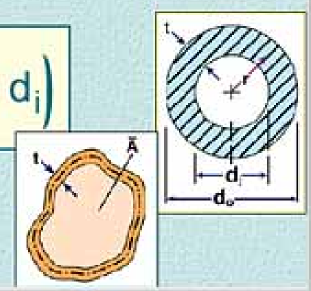
Bars with Thin-walled Closed Cross Sections

Comparison Between the Torsion Formulas for Circular Cross Section and Those Tubes

Single-Cell Tube

where

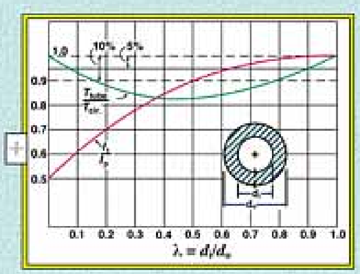
$$\oint ds = \frac{\pi}{2}\left(d_o + d_i\right)$$

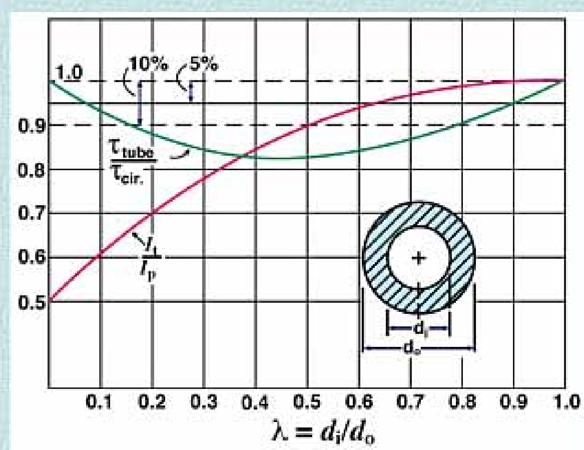


Bars with Thin-walled Closed Cross Sections

Comparison Between the Torsion Formulas for Circular Cross Section and Those Tubes

Ratios





Bars with Thin-walled Closed Cross Sections

Comparison Between the Torsion Formulas for Circular Cross Section and Those Tubes

Ratios

$$\frac{\tau_{max}\big|_{tube}}{\tau_{max}\big|_{cir}} = \frac{1+\lambda^2}{1+\lambda}$$

Bars with Thin-walled Closed Cross Sections

Comparison Between the Torsion Formulas for Circular Cross Section and Those Tubes

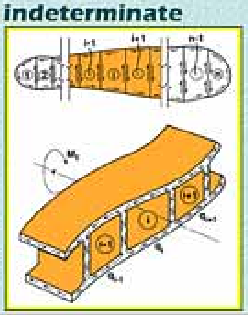
Ratios

$$\lambda = \frac{d_o}{d_i}$$

$$\frac{I_t}{I_p} = \frac{1}{2}\left(1 + \frac{2\lambda}{1+\lambda^2}\right)$$

Torsion of Multicell Thin-walled Tubes

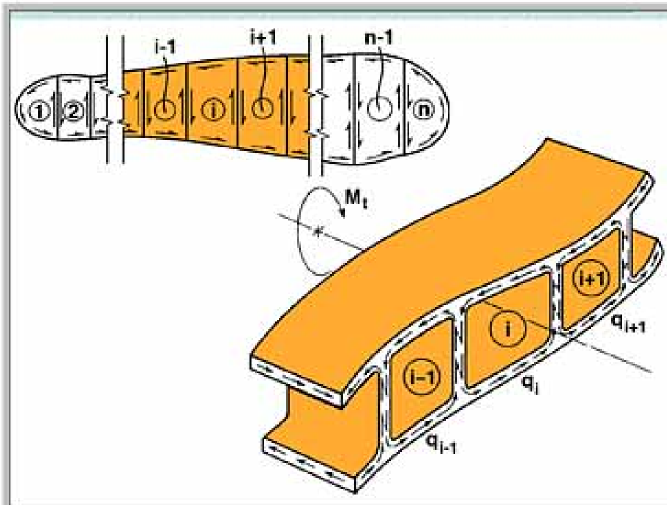
- For multicell tubes in pure torsion, equilibrium equations are not sufficient for determining the shear stresses and shear flow. Consideration of the compatibility of deformation is required to solve the problem-statically indeterminate problem.
- Consider a multicell tube with n cells.



Equilibrium

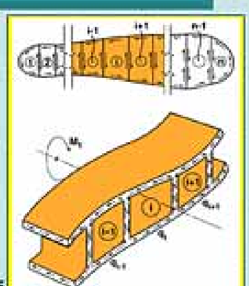
$$M_t = 2\sum_{i=1}^{n} q_i \bar{A}_i$$

where
q_i = shear flow in cell i
and $\bar{A}_i$ = area of cell i



Torsion of Multicell Thin-walled Tubes

Deformation



Cross sections warp, but do not distort in their own plane.
Entire cross section, and each cell rotate at the same rate of twist (compatibility equations)

$$\theta_1 = \theta_2 = \ldots = \theta_i = \ldots = \theta_n = \theta$$

where θ_i = rate of twist of cell i

If cell i is bounded by cells i-1 and i+1, then

$$\theta_i = \frac{1}{2G\bar{A}_i}\left[q_i \oint_{s_i} \frac{ds}{t} - q_{i-1}\int_{s_{i-1,i}} \frac{ds}{t} - q_{i+1}\int_{s_{i,i+1}} \frac{ds}{t}\right]$$

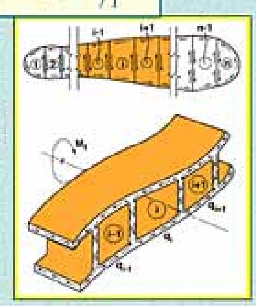
Torsion of Multicell Thin-walled Tubes

If cell i is bounded by m cells instead of two

$$\theta = \frac{1}{2G\bar{A}_i}\left[q_i \oint_{s_i} \frac{ds}{t} - \sum_{r=1}^{m}\left(q_r \int_{s_{r,i}} \frac{ds}{t}\right)\right]$$

This equation can be written in the following form:

$$f_{ii}\, q_i + \sum_{r=1}^{m} f_{ri}\, q_r - 2\bar{A}_i\,\theta = 0$$



Torsion of Multicell Thin-walled Tubes

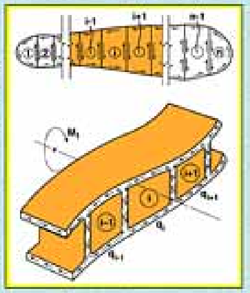
This equation can be written in the following form:

$$f_{ii}\, q_i + \sum_{r=1}^{m} f_{ri}\, q_r - 2\bar{A}_i\, \theta = 0$$

where

$$f_{ii} = \frac{1}{G} \oint_{s_i} \frac{ds}{t}$$

$$f_{ri} = \frac{-1}{G} \int_{s_{ri}} \frac{ds}{t}$$



f_{ii} and f_{ri} are called warping flexibilities.

Torsion of Multicell Thin-walled Tubes

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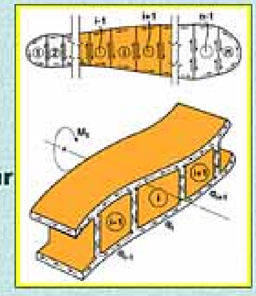
Flexibility coefficients

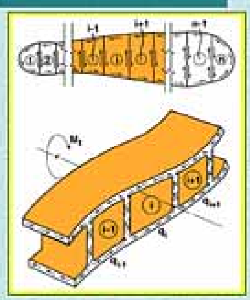
$$F_{ii} = \frac{1}{2\bar{A}_i} f_{ii} = \frac{1}{2G\bar{A}_i} \oint_{s_i} \frac{ds}{t}$$

= rate of twist due to unit shear flow in cell i

$(q_i = 1, q_r = 0, r \neq i)$



Torsion of Multicell Thin-walled Tubes



$$F_{ri} = \frac{-1}{2\bar{A}_i} f_{ri} = \frac{-1}{2G\bar{A}_i} \int_{s_{r,i}} \frac{ds}{t}$$

= rate of twist due to unit shear flow along the web between cells r and i

$(q_r = 1, q_i = 0, i \neq r)$

Bars with Hybrid Cross Section

Examples

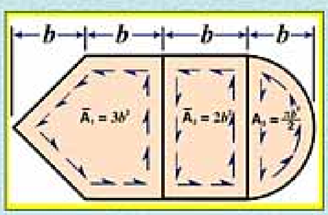
Three Cell Tube

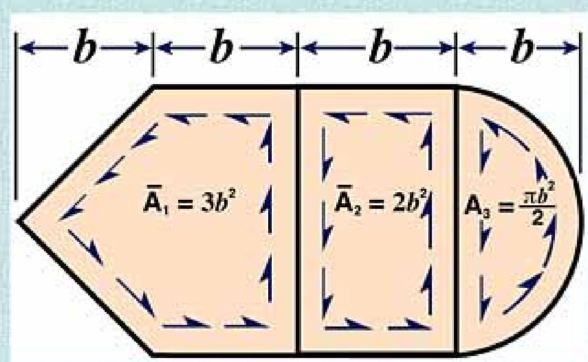
Cell Areas

$$\bar{A}_1 = 3b^2$$

$$\bar{A}_2 = 2b^2$$

$$\bar{A}_3 = \frac{\pi b^2}{2}$$



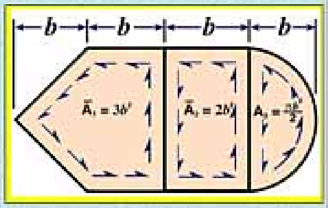


Examples

Three Cell Tube

Direct and Cross Warping Flexibilities

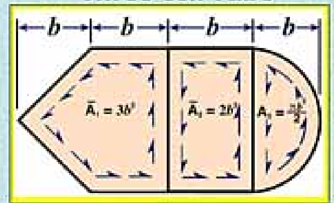
$$f_{ii} = \frac{1}{G}\oint_{ds} \frac{ds}{t}$$



$$f_{23} = f_{32} = -\frac{2b}{Gt}$$

Examples

Three Cell Tube

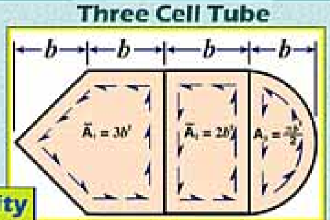


Equilibrium Equation

$$M_t = 2\left[q_1\left(3b^2\right) + q_2\left(2b^2\right) + q_3\left(\frac{\pi b^2}{2}\right)\right]$$

Examples

Three Cell Tube

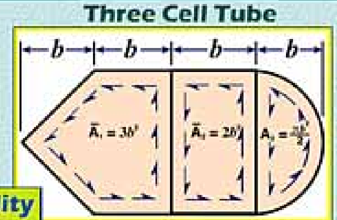


Compatibility Equations

$$\frac{1}{Gt}\left[2b\left(2 + \sqrt{2}\right) q_1 - 2b\, q_2\right] - 2\left(3b^2\right)\theta = 0$$

Examples

Three Cell Tube

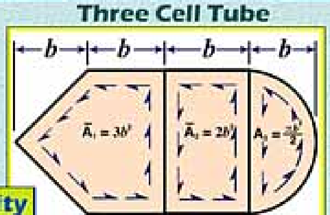


Compatibility Equations

$$\frac{1}{Gt}\left[6b\, q_2 - 2b\, q_1 - 2b\, q_3\right] - 2\left(2b^2\right)\theta = 0$$

Examples

Three Cell Tube

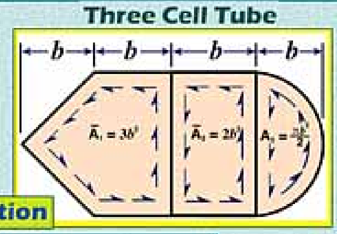


Compatibility Equations

$$\frac{1}{Gt}\left[b\left(\pi + 2\right) q_3 - 2b\, q_2\right] - 2\left(\frac{\pi b^2}{2}\right)\theta = 0$$

Examples

Three Cell Tube



Solution

$$\begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix} = \begin{Bmatrix} 0.0746 \\ 0.0851 \\ 0.0676 \end{Bmatrix} \frac{M_t}{b^2}$$

$$\theta = 0.0566\, \frac{M_t}{Gt\, b^3}$$

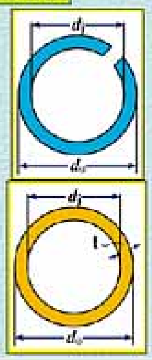
Examples

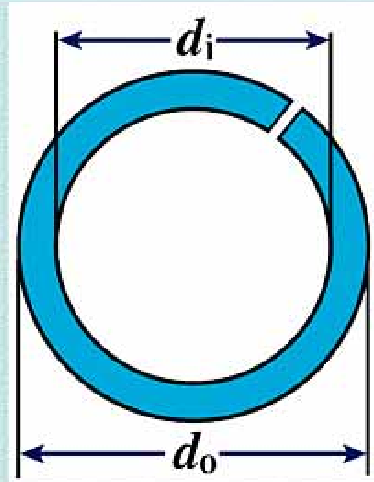
Comparison Between Torsional Stiffnesses of a Seamless and a Split Circular Thin Tube

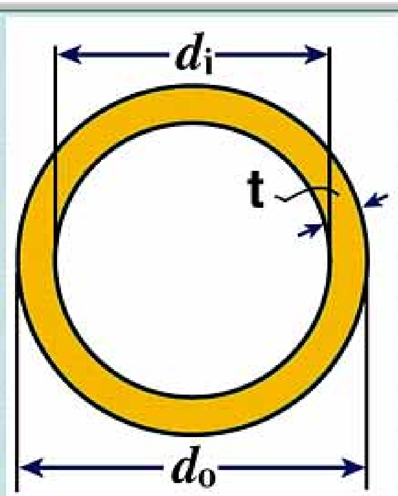
Seamless Tube

$$I_p = \frac{\pi}{32}\left(d_o^4 - d_i^4\right)$$

$$= \frac{\pi}{32}\, d_o^4\left[1 - \lambda^4\right]$$







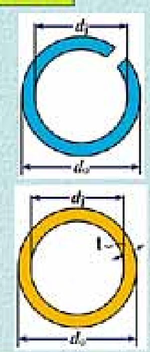
Examples

Comparison Between Torsional Stiffnesses of a Seamless and a Split Circular Thin Tube

Seamless Tube

where

$$\lambda = \frac{d_i}{d_o}$$



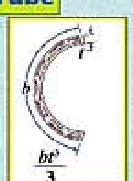
Examples

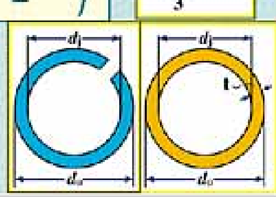
Comparison Between Torsional Stiffnesses of a Seamless and a Split Circular Thin Tube

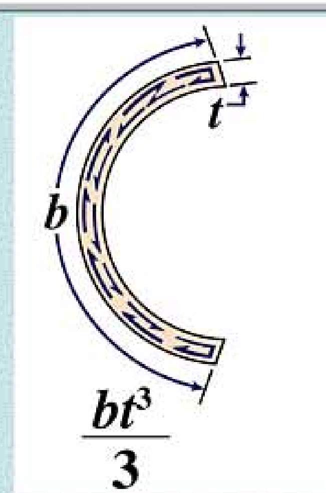
Split Tube

$$I_t = \frac{1}{3}\,\pi\left(\frac{d_o + d_i}{2}\right)\left(\frac{d_o - d_i}{2}\right)^3$$

$$\frac{I_t}{I_p} = \frac{2}{3}\,\frac{\left(d_o + d_i\right)^2}{d_o^2 + d_i^2}$$





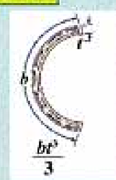


Examples

Comparison Between Torsional Stiffnesses of a Seamless and a Split Circular Thin Tube

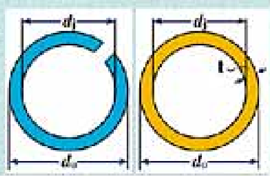
Split Tube

$$\frac{2}{3}\frac{\left(1-\lambda\right)^{2}}{1+\lambda^{2}}$$



For

$$\lambda = 0.9$$



Examples

Comparison Between Torsional Stiffnesses of a Seamless and a Split Circular Thin Tube

$$\frac{I_t}{I_p} = \frac{1}{271.5}$$